Vanguard Mathematics Department

Assessment Continuum Proposal
Draft 1

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Introduction

The Math Assessment Continuum scale is modeled after the Dreyfus Model of Skill Acquisition.

The Dreyfus model posits that in the acquisition and development of a skill, a student passes through five levels of proficiency: novice, advanced beginner, competent, proficient, and expert. These different levels reflect changes in three general aspects of skilled performance:

1. One is a movement from reliance on abstract principles to the use of past concrete experience as paradigms.
2. The second is a change in the learner's perception of the demand situation, in which the situation is seen less and less as a compilation of equally relevant bits, and more and more as a complete whole in which only certain parts are relevant.
3. The third is a passage from detached observation to involved performer. The performer no longer stands outside the situation but is now engaged in the situation.

- **Stage 1: Novice**

  Beginners have had no experience of the situations in which they are expected to perform. Novices are taught rules to help them perform. The rules are context-free and independent of specific cases; hence the rules tend to be applied universally. The rule-governed behavior typical of the novice is extremely limited and inflexible. As such, novices have no "life experience" in the application of rules. "*Just tell me what I need to do and I'll do it.*"

- **Stage 2: Advanced Beginner**

  Advanced beginners are those who can demonstrate marginally acceptable performance, those who have had experience with enough mathematical situations to recognize recurring meaningful mathematical concepts. These concepts require prior experience in actual mathematical situations for recognition. Principles to guide thinking and actions begin to be formulated. The principles are based on experience.

- **Stage 3: Competent**

  Competence, develops when the student establishes a perspective, and his/her thinking is based on considerable conscious, abstract, analytic contemplation of the problem. The conscious, deliberate planning that is characteristic of this skill level helps achieve individual and critical thinking. The competent math student lacks the aptitude and flexibility of the proficient one but does have a feeling of mastery. The competent person does not yet have enough experience to recognize a situation in terms of an overall picture or in terms of which aspects are most salient.

- **Stage 4: Proficient**

  The proficient performer perceives situations as wholes rather than in terms of chopped up parts or aspects, and performance is guided by personalized maxims. Proficient math students understand a situation as a whole because they perceive its meaning in terms of whole-picture goals. The proficient math student learns from experience what typical situations to expect in a given situation and how problem-solving needs to be modified in response to these situations.

- **Stage 5: The Expert**

  The expert performer no longer relies solely on an analytic principle (rule, guideline, maxim) to connect her or his understanding of the situation to an appropriate action. The expert performer, with an enormous background of experience, now has a deep grasp of each situation and zeroes in on the accurate region of the problem without wasteful consideration of a large range of unfruitful, alternative processes and solutions. The expert operates from a deep understanding of the total situation. The chess master, for instance, when asked why he or she made a particularly masterful move, will just say: "Because it felt right; it looked good." The performer is no longer consciously aware of features and rules that she is using; her performance becomes fluid and flexible and highly proficient. This is not to say that the expert never uses analytic tools. Highly skilled analytic ability is necessary for those situations with which the performer has had no previous experience. Analytic tools are also necessary for those times when the expert gets a wrong grasp of the situation and then finds that results are not occurring as expected. When alternative perspectives are not available to the performer, the only way out of a wrong grasp of the problem is by using analytic problem solving.

Asking "What If?"/Conjecture

Put an 'X' where you think the performer is on the continuum, then provide explanation and evidence to support your assessment.

One case — *Can Generalize*

Novice	Advanced Beginner	Competent	Proficient	Expert
Asks questions about mathematical procedures, theorems, rules	*Develops conjecture/ hypothesis/claim about an observation or pattern*	*Tests conjectures and proves that conjecture is true or false*	*Tests conjectures and begins to make generalizations for other scenarios. Develops connections.*	*Tests conjectures and generalizes for a multitude of scenarios. Makes connections easily. Sophisticated proofs.*

Assessment Evidence:

<u>Generalization: Exploration, Generalization and Conjecture</u>

Before a mathematical fact can be proved, it must be articulated – the conjecture phase of the explore-generalize-conjecture-prove reasoning path common in mathematics. A conjecture represents a mathematical claim about a generalization or abstraction.

For instance, if students draw and measure the angles of many triangles, I would want them to notice that, in general, all triangles have a total interior angle measure of 180 degrees. If students make tables of linear values, they may conjecture that the y-value at $x = 0$ will always be the y-intercept of the graph, regardless of the linear equation they are dealing with.

Given situations, students should be able to explore particular cases and find patterns from them. Once they have identified explicit or recursive patterns, they should be able to generalize from them and explain how to think of the problem in abstract, generalized terms. After this, students should ideally be able to explain *why* their abstraction makes sense – they should be able to produce proof or, if they cannot prove their claims, provide a counter-example. We as instructors must also be diligent and wary of the risk of over-generalization, a common source of misconceptions.

Making Connections – *NCTM: Connections*

Put an 'X' where you think the performer is on the continuum, then provide explanation and evidence to support your assessment.

Obvious ←→ *Sophisticated*

Novice	Advanced Beginner	Competent	Proficient	Expert
Recognizing material studied previously ("this is the same as what we did…") *Connects problem-solving methods to each other ("my team solved it the same way, using elimination")*	*algebra. Connects pictures to connections among Connects different representations to each other (graph to table to algebra)*	*Recognizes and uses applies mathematics mathematical ideas*	*Recognizes and mathematical ideas in contexts outside mathematics*	*Understands how* *interconnect and build on one another to produce a coherent whole*

Assessment Evidence:

<u>Connections: Connecting, Comparing and Contrasting Ideas and Relationships</u>

Students should be able to explain what types of variables a situation requires and how those variables interact with each other. Students should also be able to conjecture about what types of mathematical relationships apply to their situation, and they should be able to connect those relationships into broader functional-families, which include:

- direct / proportional
- linear
- inverse proportional / hyperbolic
- inverse-squared
- logarithmic
- trigonometric
- quadratic
- n^{th}-degree polynomial
- exponential
- rational functions

Students should also be able to make connections among problems. For instance, students should be able to explain how a given problem connects to ones they have previously solved, and what things are different. For instance, students should be able to link together:

- modeling the ratio of triangle side lengths as function of an angle,
- modeling the depth of tidewater as a function of the time of day, and
- modeling the potential and kinetic energy of pendulum as a function of position.

The must also be able to explain why the same model makes sense in these situations.

Finally, students should appreciate that pieces of content that may seem disparate on the surface actually harbor deep connections. For instance, the distance formula, the Pythagorean Theorem, the analytical definition of a circle and the trigonometric identity $\sin^2 x + \cos^2 x = 1$ are, at their core, all fundamentally connected.

Being Metacognitive

Put an 'X' where you think the performer is on the continuum, then provide explanation and evidence to support your assessment.

Superficial → *Reflective*

Novice	Advanced Beginner	Competent	Proficient	Expert
Explains why mathematical moves are used. *Asks questions about what s/he or group is doing*	*Recognizes when s/he does not understand and takes action to gain understanding, by asking questions of self, teammates, or teacher*	*Understands when a mistake or misconception is applied and is able to revise thinking*	*Consistent reflection and revision of one's thinking* *Makes/tests conjectures to develop understanding when lost*	*Consistent recognition of understanding as well as lack of understanding* *Makes/tests conjectures consistently, while making connections to thinking in other disciplines*

Assessment Evidence:

Seeking Significance – *NCTM: Problem-Solving*

Put an 'X' where you think the performer is on the continuum, then provide explanation and evidence to support your assessment.

One strategy → *Multiple strategies*

Novice	Advanced Beginner	Competent	Proficient	Expert
Uses a variety of problem solving strategies to understand new mathematical content	*Observes and explains patterns to formulate generalizations and conjectures*	*Uses multiple representations to explain and solve problems (verbally, numerically, graphically, algebraically)* *Work in collaboration with others to propose, critique, evaluate, and value alternate approaches*	*Evaluates the relative efficiency of different representations and solution methods of a problem* *Uses a variety of strategies to extend solution methods to other problems*	*Determines information required to solve the problem, chooses methods for obtaining the info., and defines parameters for acceptable solutions* *Interprets solutions within the given restraints of a problem*

Assessment Evidence:

<u>Seeking Significance: Problem Solving</u>

Problem solving is an integral part of all mathematics learning. In everyday life and in the workplace, being able to solve problems can lead to great advantages. However, solving problems is not only a goal of learning mathematics but also a major means of doing so. Problem solving means engaging in a task for which the solution is not known in advance. Good problem solvers have a "mathematical disposition"--they analyze situations carefully in mathematical terms and naturally come to pose problems based on situations they see. For example, a young child might wonder, How long would it take to count to a million?

Students need to develop a range of strategies for solving problems, such as using diagrams, looking for patterns, or trying special values or cases. These strategies need instructional attention if students are to learn them. However, exposure to problem-solving strategies should be embedded across the curriculum. Students also need to learn to monitor and adjust the strategies they are using as they solve a problem.

Students who are good problem-solvers will:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Evidence – *NCTM: Reasoning & Proof and Representation*

Put an 'X' where you think the performer is on the continuum, then provide explanation and evidence to support your assessment.

Simplistic reasoning ← → *Complex proof*

Novice	Advanced Beginner	Competent	Proficient	Expert
Recognizes mathematical ideas can be supported by a variety of strategies	*Investigates and evaluates conjectures in mathematical terms, using strategies to reach conclusions* *Recognizes when an approximation is more appropriate than an exact answer*	*Presents correct mathematical arguments in a variety of forms* *Provides correct mathematical arguments in response to other students' conjectures, reasoning, & arguments*	*Constructs a proof using one or two methods (deductive, analytic, transformational)* *Evaluates written arguments for validity* *Devises ways to verify or use counterexamples to refute incorrect statements*	*Constructs a proof using a variety of methods* *Devises ways to verify results, using counterexamples and informal indirect proof*

Assessment Evidence:

<u>Evidence: Reasoning, Proof, and Representation</u>

Reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts. Exploring, justifying, and using mathematical conjectures are common to all content areas and, with different levels of rigor, all grade levels. Through the use of reasoning, students learn that mathematics makes sense. Reasoning and proof must be a consistent part of student's mathematical experiences.

Representations are necessary to students' understanding of mathematical concepts and relationships. Representations allow students to communicate mathematical approaches, arguments, and understanding to themselves and to others. They allow students to recognize connections among related concepts and apply mathematics to realistic problems. Students who are good at providing reasoning and proof and representation:

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.
- create and use representations to organize, record, and communicate
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

Considering Viewpoints – *NCTM: Communication*

Put an 'X' where you think the performer is on the continuum, then provide explanation and evidence to support your assessment.

Open to others *Analytical & articulate*

Novice	Advanced Beginner	Competent	Proficient	Expert
Communicates verbally and is able to explain steps in solving a problem	*Uses mathematical representations to communicate (tables, formulate, equations, graphs, diagrams)* *Communicates logical arguments clearly, showing why a result makes sense and why the reasoning is valid*	*Can explain relationships among different represent-ations of a problem* *Supports or rejects arguments or questions raised by others*	*Reads/listens for logical understanding of mathematical thinking shared by others* *Reflects on strategies of others in relation to one's own strategy*	*Formulates mathematical questions that elicit, extend or challenge strategies, solutions, and/or conjectures of others*

Assessment Evidence:

Considering Viewpoints: Communicating and Learning from Others

As students are asked to communicate about the mathematics they are studying--to justify their reasoning to a classmate or to formulate a question about something that is puzzling--they gain insights into their thinking. In order to communicate their thinking to others, students naturally reflect on their learning and organize and consolidate their thinking about mathematics.

Students should be encouraged to increase their ability to listen to others and express themselves clearly and coherently. As they become older, their styles of argument and dialogue should more closely adhere to established conventions, and students should become more aware of, and responsive to their audience. The ability to write about mathematics should be particularly nurtured.

By working on problems with classmates, students also have opportunities to see the perspectives and methods of others. They can learn to understand and evaluate the thinking of others and to build on those ideas. They may benefit from the insights of students who solve the problem using a visual representation. Students need to learn to weigh the strengths and limitations of different approaches, thus becoming critical thinkers about mathematics.